PROPAGAZIONE GUIDATA

ESISTE STRUTTURA CHE <u>SUPPORTA</u>

LA PROPAGAZIONE

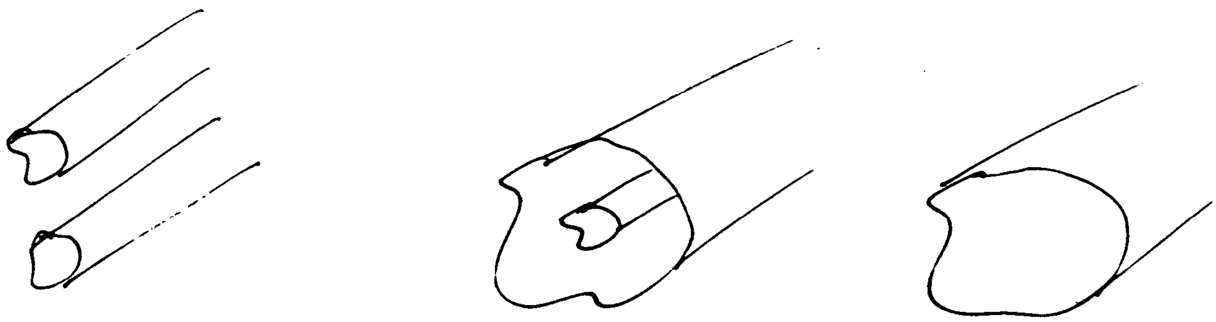
SUPERFICI CILINDRICHE INDEFINITE

DI MATERIALE CONDUTTORE ELETTRICO

PERFETTO

DIELETTRICO OMOGENEO, ISOTROTO, PRIVO DI

PERDITE



CILINDRI RETTI

RICERCA DELLE SOLUZIONI LIBERE

$$\nabla \times \underline{E} = -j\omega\mu \underline{H} \qquad \nabla \cdot \varepsilon \underline{E} = 0$$

$$\nabla \times \underline{H} = j\omega\varepsilon E \qquad \nabla \cdot \mu \underline{H} = 0$$

BASTA CONSIDERARE LE

SFRUTTIAMO LA GEOMETRIA, E PONIAMO:

$$\underline{E} = \underline{E}_t + E_z \underline{i}_z \qquad \underline{E}_t = E_x \underline{i}_x + E_y \underline{i}_y$$

$$\underline{H} = \underline{H}_t + H_z \underline{i}_z \qquad \underline{H}_t = H_x \underline{i}_x + H_y \underline{i}_y$$

Proiettiamo sul piano trasverso:

$$\underline{i}_z \times \quad \nabla \times \underline{E}_t + \nabla \times E_z \underline{i}_z = -j\omega\mu \underline{H}_t \underbrace{- j\omega\mu H_z \underline{i}_z}_{=\emptyset}$$

$$\underline{i}_z \times \quad \nabla \times \underline{H}_t + \nabla \times H_z \underline{i}_z = j\omega\varepsilon \underline{E}_t + \underbrace{j\omega\varepsilon E_z \underline{i}_z}_{=\emptyset}$$

Ricordando:

$$\underline{A} \times (\underline{B} \times \underline{C}) = \underline{B}(\underline{A} \cdot \underline{C}) - \underline{C}(\underline{A} \cdot \underline{B})$$

$$\underline{i}_z \times \nabla \times \underline{E}_t = \nabla(\underbrace{\underline{i}_z \cdot \underline{E}_t}_{=0}) - \underline{E}_t(\underline{i}_z \cdot \nabla)$$

$$\underline{i}_z \cdot \nabla = \underline{i}_z \cdot \left(\underline{i}_x \frac{\partial}{\partial x} + \underline{i}_y \frac{\partial}{\partial y} + \underline{i}_z \frac{\partial}{\partial z} \right)$$

$$= \frac{\partial}{\partial z}$$

$$\underline{i}_z \times \underbrace{\nabla \times E_z \underline{i}_z} = \underline{i}_z \times \left(\begin{vmatrix} \underline{i}_x & \underline{i}_y & \underline{i}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_z \end{vmatrix} \right) =$$

$$= \underline{i}_z \times \left(\underline{i}_x \frac{\partial E_z}{\partial y} - \underline{i}_y \frac{\partial E_z}{\partial x} \right) = \underline{i}_y \frac{\partial E_z}{\partial y} + \underline{i}_x \frac{\partial E_z}{\partial x}$$

PERTANTO:

$$-\frac{\partial}{\partial z} \underline{E}_t + \nabla_t E_z = -j\omega\mu\, \underline{i}_z \times \underline{H}_t$$

$$-\frac{\partial}{\partial z} \underline{H}_t + \nabla_t H_z = j\omega\varepsilon\, \underline{i}_z \times \underline{E}_t$$

PROIETTIAMO [redacted] LONGITUDINALMENTE:

$$\underline{i}_z \cdot \qquad \nabla \times \underline{E}_t + \underset{\overset{\parallel}{0}}{\nabla \times E_z \underline{i}_z} = -j\omega\mu \underset{\overset{\parallel}{0}}{\underline{H}_t} - j\omega\mu H_z \underline{i}_z$$

$$\underline{i}_z \cdot \qquad \nabla \times \underline{H}_t + \nabla \times H_z \underline{i}_z = j\omega\varepsilon \underline{E}_t + j\omega\varepsilon E_z \underline{i}_z$$

$$\underline{i}_z \cdot \nabla \times \underline{E}_t = -j\omega\mu H_z \rightarrow H_z = -\frac{1}{j\omega\mu} \nabla_t \cdot \underline{E}_t \times \underline{i}_z$$

$$\begin{cases} H_z = \dfrac{1}{j\omega\mu} \nabla_t \cdot \left(\underline{i}_z \times \underline{E}_t \right) \\ E_z = \dfrac{1}{j\omega\varepsilon} \nabla_t \cdot \left(\underline{H}_t \times \underline{i}_z \right) \end{cases}$$

E QUINDI:

$$\begin{cases} -\dfrac{\partial}{\partial z} \underline{E}_t = j\omega\mu \left[\left(\underline{H}_t \times \underline{i}_z \right) + \dfrac{1}{k^2} \nabla_t \nabla_t \cdot \left(\underline{H}_t \times \underline{i}_z \right) \right] \\ -\dfrac{\partial}{\partial z} \underline{H}_t = j\omega\varepsilon \left[\left(\underline{i}_z \times \underline{E}_t \right) + \dfrac{1}{k^2} \nabla_t \nabla_t \cdot \left(\underline{i}_z \times \underline{E}_t \right) \right] \end{cases}$$

$$E_z =$$

$$H_z =$$

Poniamo:

$$\underline{E}_t = \underline{e}(\underline{t})\, V(z) \qquad \underline{t} = x\,\underline{i}_x + y\,\underline{i}_y$$

$$\underline{H}_t = \underline{h}(\underline{t})\, I(z)$$

$\underline{e}(\underline{t})$, $\underline{h}(\underline{t})$ → funzioni vettoriali di modo

$V(z)$, $I(z)$ → " scalari di modo

Studiamo alcune configurazioni:

(TEM) $\qquad E_z = H_z = 0$

$$\underline{E} = \underline{E}_t \qquad \underline{H} = \underline{H}_t$$

(TE) $\qquad E_z = 0$

$$\underline{E} = \underline{E}_t \qquad \underline{H} = \underline{H}_t + H_z\,\underline{i}_z$$

(TM) $\qquad H_z = 0$

$$\underline{E} = \underline{E}_t + E_z\,\underline{i}_z \qquad \underline{H} = \underline{H}_t$$

DEFINISCONO QUALSIASI CONFIGURAZIONE
(SI VEDRA')

TEM

$$E_z = H_z = \emptyset$$

$$\underline{E} = \underline{E}_t \qquad \underline{H} = \underline{H}_t$$

$$\begin{cases} -\dfrac{\partial}{\partial z}\underline{E}_t = j\omega\mu\,(\underline{H}_t \times \underline{i}_z) \\ -\dfrac{\partial}{\partial z}\underline{H}_t = j\omega\varepsilon\,(\underline{i}_z \times \underline{E}_t) \end{cases} \rightarrow \begin{cases} -\underline{e}\dfrac{dV}{dz} = j\omega\mu I\,(\underline{h}\times\underline{i}_z) \\ -\underline{h}\dfrac{dI}{dz} = j\omega\varepsilon V\,(\underline{i}_z \times \underline{e}) \end{cases}$$

$$\underline{e} = \frac{j\omega\mu I}{-\dfrac{dV}{dz}}(\underline{h}\times\underline{i}_z) \rightarrow \underline{e} = a\,(\underline{h}\times\underline{i}_z)$$

$$\underline{i}_z \times \underline{e} = a\;\underline{i}_z \times (\underline{h}\times\underline{i}_z) = a\,\underline{h}$$

$$\boxed{\underline{e} = a\,(\underline{h}\times\underline{i}_z) \qquad \underline{h} = \frac{1}{a}(\underline{i}_z \times \underline{e})}$$

$$\underline{h} = \frac{j\omega\varepsilon V}{-\dfrac{dI}{dz}}(\underline{i}_z \times \underline{e})$$

$$\frac{j\omega\mu I}{-\frac{dV}{dz}} = a \longrightarrow \boxed{-\frac{dV}{dz} = j\omega\mu\frac{1}{a}I}$$

$$\frac{j\omega\varepsilon V}{-\frac{dI}{dz}} = \frac{1}{a} \longrightarrow \boxed{-\frac{dI}{dz} = j\omega\varepsilon a V}$$

da cui si ricava V, I

ed $\underline{e}, \underline{h}$?

$$E_z = \emptyset \quad \nabla_t \cdot (\underline{H}_t \times \underline{i}_z) = \emptyset \longrightarrow \nabla_t \times \underline{H}_t = \emptyset$$

$$H_z = \emptyset \quad \nabla_t \cdot (\underline{i}_z \times \underline{E}_t) = \emptyset \longrightarrow \nabla_t \times \underline{E}_t = \emptyset$$

Ricordando: $\nabla \cdot (\underline{a} \times \underline{b}) = \underline{b} \cdot \nabla \times \underline{a} - \underline{a} \cdot \nabla \times \underline{b}$

$$I(z)\, \nabla_t \times \underline{h} = 0 \qquad \nabla_t \times \underline{h} = 0 \qquad \underline{h} = -\nabla_t \psi$$

$$V(z)\, \nabla_t \times \underline{e} = 0 \qquad \nabla_t \times \underline{e} = 0 \qquad \underline{e} = -\nabla_t \phi$$

Sempre da:

$$\begin{cases} \nabla_t \cdot (\underline{H}_t \times \underline{i}_z) = 0 \longrightarrow I(z)\, \nabla_t \cdot (\underline{h} \times \underline{i}_z) = \emptyset & \nabla_t \cdot \underline{e} = \emptyset \\ \nabla_t \cdot (\underline{i}_z \times \underline{E}_t) = \emptyset \longrightarrow V(z)\, \nabla_t \cdot (\underline{i}_z \times \underline{e}) = 0 & \nabla_t \cdot \underline{h} = \emptyset \end{cases}$$

$$\nabla_t \cdot \underline{e} = -\nabla_t \cdot \nabla_t \phi = -\nabla_t^2 \phi = 0$$

$$\nabla_t \cdot \underline{h} = -\nabla_t \cdot \nabla_t \psi = -\nabla_t^2 \psi = 0$$

$$\boxed{\nabla_t^2 \phi = 0 \qquad \nabla_t^2 \psi = 0}$$

$\phi, \psi \rightarrow$ funzioni armoniche

$$\underline{e} = -\nabla_t \phi \qquad\qquad \underline{h} = -\nabla_t \psi$$

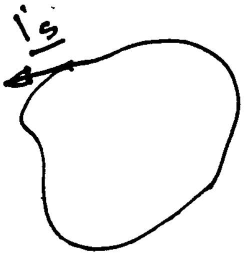
$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\underline{h} = \frac{1}{a}(\underline{i}_z \times \underline{e}) \qquad\qquad \underline{e} = (\underline{h} \times \underline{i}_z)\, a$$

$$\nabla^2 \phi = 0 \qquad \text{con } \phi\Big]_{contorno} = \, ?$$

$$\underline{E} \cdot \underline{i_s}\Big]_C = \emptyset = \underline{E_t} \cdot \underline{i_s}\Big]_C = V\, \underline{e} \cdot \underline{i_s}\Big]_C = \emptyset$$

$$\nabla_t \phi \cdot \underline{i_s}\Big] = \emptyset = \frac{\partial \phi}{\partial s}\Big]_C = \emptyset$$



$$\boxed{\phi_C = \text{cost}}$$

Una funzione armonica costante sul contorno, se la sezione è semplicemente connessa, mantiene lo stesso valore costante su tutta la sezione.

$$\underline{e} = -\nabla\phi = -\nabla \text{cost.} = \emptyset$$

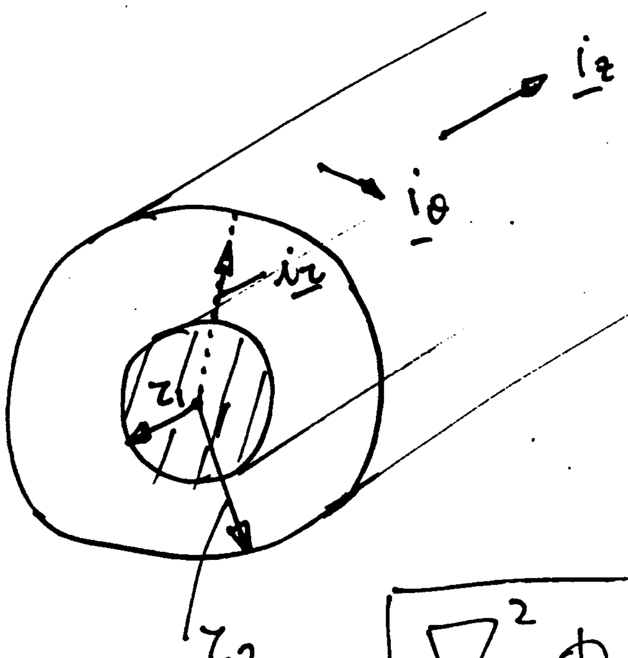
Quindi campi nulli.

Se la sezione è moltepliciemente connessa:

$$\phi\Big]_{Ci} = \text{cost.} \quad \text{e quindi} \quad \underline{e} \neq \emptyset$$

Esistenza di modo TEM

Esempio: TEM in cavo coassiale.



sezione duplicemente connessa

$$\boxed{\nabla_t^2 \phi = 0}$$

$$\phi = \phi(\underline{t}) = \phi(r, \theta)$$

$$\phi(r_1, \theta) = \phi_1 \quad \text{(costante)}$$

$$\phi(r_2, \theta) = \phi_2 \quad (\quad " \quad)$$

$$\boxed{\phi = \phi(r)}$$

$$\nabla_t^2 \phi = \frac{1}{r} \frac{d}{dr}\left(r \frac{d\phi}{dr}\right) = 0 \qquad \boxed{r_1 \leq r \leq r_2}$$

$$\downarrow$$

$$\frac{d}{dr}\left(r\frac{d\Phi}{dr}\right)=0 \rightarrow r\frac{d\Phi}{dr}=-A$$

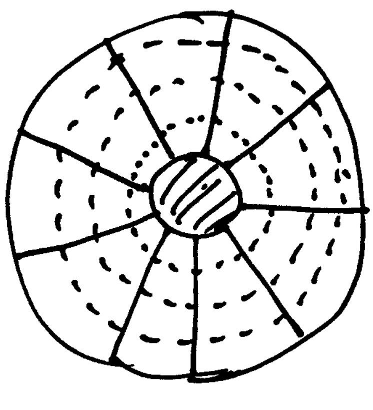
$$d\Phi=-\frac{A}{r}dr$$

$$\Phi(r)=-A\lg r+C$$

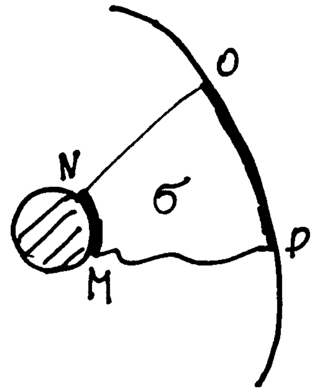
$$\underline{e}=-\nabla\Phi=-\frac{\partial\Phi}{\partial r}\underline{i_r}=\frac{A}{r}\underline{i_r}$$

$$\underline{h}=\frac{1}{a}\underline{i_z}\times\underline{e}$$

$$\boxed{\begin{cases}\underline{E}=\underline{E}_t=\underline{e}V=\dfrac{A}{r}\underline{i_r}V(z)\\[2ex]\underline{H}=\underline{H}_t=\underline{h}I=\dfrac{1}{a}\dfrac{A}{r}\underline{i_\theta}I(z)\end{cases}}$$



Fissiamo A, a in modo da attribuire a V, I significato fisico.



$$\nabla \times \underline{E} = -j\omega\mu \underline{H}$$

$$\iint_\sigma \nabla \times \underline{E}_t \cdot \underline{i}_z \, d\sigma = -j\omega\mu \iint_\sigma \underline{H}_t \cdot \underline{i}_z \, d\sigma$$

$$\oint_{MNOP} E_t \cdot \underline{i}_s \, ds = \emptyset$$

$$\int_M^N \cancel{\quad} + \int_N^O \cdots\cdots + \int_O^P \cancel{\cdots\cdots} + \int_P^M \cdots\cdots = \emptyset$$

$$\int_N^O \underline{E}_t \cdot \underline{i}_s \, ds = \int_M^P \underline{E}_t \cdot \underline{i}_s \, ds =$$ tensione o differenza di potenziale

$$\int_N^O \underline{E}_t \cdot \underline{i}_s \, ds = \int_{r_1}^{r_2} \frac{A}{r} V(z) \, \underline{i}_r \cdot \underline{i}_r \, dr =$$

$$= A\,V(z) \lg \frac{r_2}{r_1} = V(z)$$ alla $V(z)$ si dà il significato di...!

da cui $A = \frac{1}{\lg \frac{r_2}{r_1}}$

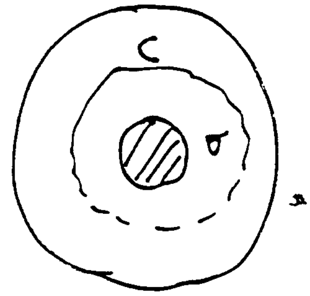
e quindi:

$$\boxed{\underline{E}_t = \underline{E} = \frac{V(z)}{r \lg \frac{r_2}{r_1}} \, \underline{i_r}}$$

DALLA 2ª EQ. DI MAXWELL:

$$\nabla \times \underline{H} = j\omega\varepsilon \underline{E} + \underline{J}$$

$$\iint_\sigma \nabla \times \underline{H}_t \cdot \underline{i}_z \, d\sigma = j\omega\varepsilon \underbrace{\iint_\sigma \underline{E}_t \cdot \underline{i}_z \, d\sigma}_{=\emptyset} + \iint_\sigma \underline{J} \cdot \underline{i}_z \, d\sigma$$



$$\oint_c \underline{H}_t \cdot \underline{i}_c \, dc = I_c(z)$$

dove $I_c(z)$ è la corrente nel conduttore connessa con il campo $\underline{H}_t$; fuori del conduttore la corrente è nulla

Se C è una circonferenza di raggio r generico

Si ha:

$$\boxed{\int_0^{2\pi} \left(\frac{1}{a}\frac{A}{r} I(z)\, \underline{i}_\theta\right) \cdot \left(\underline{i}_\theta\, r\, d\theta\right) = I_c(z)}$$

funzione scalare di modo

corrente nel conduttore I_c

Se poniamo che siano uguali: $I_c = I(z)$

$$\frac{1}{a}\frac{A}{\cancel{r}} I(z) \cancel{r}\, 2\pi = I(z)$$

$$a = 2\pi A = \frac{2\pi}{\lg \frac{r_2}{r_1}}$$

e quindi
$$\boxed{\underline{H}_t = \underline{H} = \frac{1}{2\pi r} I(z)\, \underline{i}_\theta}$$

$V \to$ dimensioni di una tensione [Volt]

$I \to$ " " " corrente [A]

$\underline{e} \equiv [L^{-1}]$; $\underline{h} \equiv [L^{-1}]$

Si è trovato: a, A

$$\begin{cases} -\dfrac{dV}{dz} = j\omega\mu \dfrac{1}{a} I \\ -\dfrac{dI}{dz} = j\omega\varepsilon a V \end{cases}$$

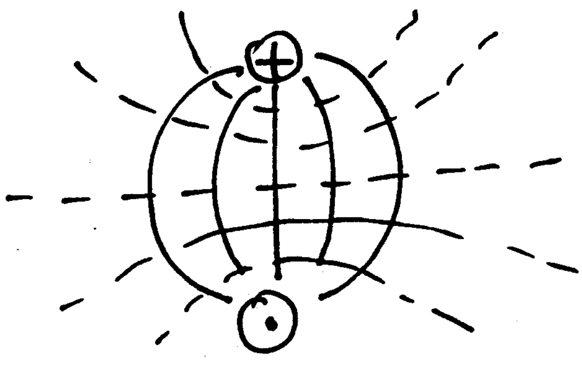
Nella teoria delle linee, si era trovato:

$$\begin{cases} -\dfrac{dV}{dz} = j\omega L I \\ -\dfrac{dI}{dz} = j\omega C V \end{cases}$$

Cavo coassiale $\equiv$ linea di trasmissione

$$C = \frac{2\pi\varepsilon}{\lg \frac{r_2}{r_1}} \qquad L = \frac{\mu}{2\pi} \lg \frac{r_2}{r_1}$$

C, L per unità di lunghezza -



$\boxed{\text{Modi TE}}$ $\quad E_z = 0$

$$\underline{E} = \underline{E}_t = \underline{e}(t)\,V(z)$$

$$\underline{H} = \underline{H}_t + H_z \underline{i}_z = \underline{h}\,I(z) + H_z \underline{i}_z$$

Eq. trasv.

$$\begin{cases} -\dfrac{\partial \underline{E}_t}{\partial z} = j\omega\mu \left[\left(\underline{H}_t \times \underline{i}_z\right) + \dfrac{1}{k^2} \nabla_t \nabla_t \cdot \left(\underline{H}_t \times \underline{i}_z\right) \right] \\ -\dfrac{\partial \underline{H}_t}{\partial z} = j\omega\varepsilon \left[\left(\underline{i}_z \times \underline{E}_t\right) + \dfrac{1}{k^2} \nabla_t \nabla_t \cdot \left(\underline{i}_z \times \underline{E}_t\right) \right] \end{cases}$$

$$\begin{cases} E_z = \dfrac{1}{j\omega\varepsilon} \nabla_t \cdot \left(\underline{H}_t \times \underline{i}_z\right) \longrightarrow \boxed{\nabla_t \cdot \left(\underline{H}_t \times \underline{i}_z\right) = 0} \\ H_z = \dfrac{1}{j\omega\mu} \nabla_t \cdot \left(\underline{i}_z \times \underline{E}_t\right) \end{cases}$$

e quindi:

$$\begin{cases} -\underline{e}\,\dfrac{dV}{dz} = j\omega\mu\, I \left(\underline{h} \times \underline{i}_z\right) \\ -\underline{h}\,\dfrac{dI}{dz} = j\omega\varepsilon\, V \left[\left(\underline{i}_z \times \underline{e}\right) + \dfrac{1}{k^2} \nabla_t \nabla_t \cdot \left(\underline{i}_z \times \underline{e}\right) \right] \end{cases}$$

$$\underline{e} = \underbrace{\frac{j\omega\mu I}{-\dfrac{dV}{dz}}}_{=a} \left(\underline{h} \times \underline{i}_z\right)$$

$$\begin{cases} -\dfrac{dV}{dz} = \dfrac{1}{a}\, j\omega\mu I \\ \underline{e} = a \left(\underline{h} \times \underline{i}_z\right) \end{cases}$$

- arbitrarietà nella scelta di a
- impossibilità di associare significato fisico a V ed I
- poniamo $a = 1$

quindi: $\underline{e} = (\underline{h} \times \underline{i}_z)$ $\quad \underline{i}_z \times \underline{e} = \underline{i}_z \times \underline{h} \times \underline{i}_z = \underline{h}$

pertanto:

$$-\underline{h}\frac{dI}{dz} - j\omega\varepsilon V \underline{h} = j\frac{\omega\varepsilon V}{k^2}\nabla_t \nabla_t \cdot \underline{h}$$

$$\downarrow$$

$$\nabla_t \nabla_t \cdot \underline{h} = \frac{k^2}{j\omega\varepsilon V}\left(-\frac{dI}{dz} - j\omega\varepsilon V\right)\underline{h}$$

$$\downarrow$$

$$\nabla_t \nabla_t \cdot \underline{h} = -\underbrace{k^2\left(\frac{dI/dz}{j\omega\varepsilon V} + 1\right)}_{\text{cost}}\underline{h}$$

$$\begin{cases} k^2\left(\dfrac{dI/dz}{j\omega\varepsilon V} + 1\right) = \text{cost} = k_{tH}^2 \\ \nabla_t \nabla_t \cdot \underline{h} + k_{tH}^2\,\underline{h} = 0 \end{cases}$$

$$\frac{\cancel{\omega^2}\varepsilon\mu}{j\cancel{\omega}\varepsilon V}\frac{dI}{dz} + k^2 = k_{tH}^2$$

$$\boxed{\begin{array}{l} -\dfrac{dI}{dz} = j\,\dfrac{k^2 - k_{tH}^2}{\omega\mu}\,V \\ \nabla_t \nabla_t \cdot \underline{h} + k_{tH}^2\,\underline{h} = 0 \\ \hline -\dfrac{dV}{dz} = j\omega\mu\, I \\ \hline \underline{e} = \underline{h} \times \underline{i}_z \end{array}}$$

poniamo

$$k_{zH}^2 = k^2 - k_{tH}^2$$

e valutiamo:

$$\frac{k_{zH}}{\omega\mu} \equiv \left[\frac{\omega\sqrt{\varepsilon\mu}}{\omega\mu}\right] \equiv \left[\sqrt{\frac{\varepsilon}{\mu}}\right] \equiv \left[\frac{1}{Z}\right]$$

e quindi:

$$\frac{k_{zH}}{\omega\mu} = \frac{1}{Z_{0H}} \rightarrow \omega\mu = k_{zH} Z_{0H}$$

$$\begin{cases} -\dfrac{dI}{dz} = j\,\dfrac{k_{zH}}{Z_{0H}}\,V \\ -\dfrac{dV}{dz} = j\,k_{zH} Z_{0H}\, I \end{cases} \longrightarrow \text{noto } k_{zH} \rightarrow \underline{V},\ I$$

In ogni caso è:

$$kZ = \omega\sqrt{\varepsilon\mu}\sqrt{\frac{\mu}{\varepsilon}} = \omega\mu$$

e quindi anche per il modo TEM, è possibile usare le stesse notazione.

Risolviamo:

$$\nabla_t \nabla_t \cdot \underline{h} + k_{tH}^2 \underline{h} = \emptyset \qquad \text{da } E_z = \emptyset$$

$$\downarrow$$

$$\nabla_t \cdot (\underline{H}_t \times \underline{i}_z) = \emptyset$$

$$\nabla_t \times \underline{h} = \emptyset \qquad \underline{h} = -\nabla_t \psi$$

e quindi:

$$-\nabla_t \nabla_t \cdot \nabla_t \psi - k_{tH}^2 \nabla_t \psi = \emptyset$$

$$\nabla_t \left(\nabla_t \cdot \nabla_t \psi + k_{tH}^2 \psi \right) = \emptyset$$

$$\downarrow$$

$$\nabla_t^2 \psi + k_{tH}^2 \psi = \text{cost}$$

$$\psi = \psi_0 + \psi_1$$

$$\downarrow$$

omogenea corrispondente

$$\psi_1 = \frac{\text{cost}}{k_{tH}^2}$$

$$\underline{h} = -\nabla_t \psi = -\nabla_t \psi_0 - \nabla_t \psi_1 = -\nabla_t \psi_0$$

Soluzione:

$$\nabla_t^2 \psi + k_{tH}^2 \psi = \emptyset$$

con le condizione al contorno:

$$\underline{E}\cdot\underline{i}_c\Big]_c = \emptyset = \underline{E}_t \times \underline{i}_n\Big]_c = \emptyset = V\underline{e}\times\underline{i}_n\Big]_c = \underline{e}\times\underline{i}_n\Big]_c = \emptyset$$

$$\underline{i}_n \times \underline{h} \times \underline{i}_z\Big]_c = \underline{h}\,\cancel{(\underline{i}_n\cdot\underline{i}_z)} - \underline{i}_z(\underline{i}_n\cdot\underline{h})\Big]_c = \emptyset$$

$$\underline{i}_n\cdot\underline{h}\Big]_c = \emptyset = \underline{i}_n\cdot\nabla_t\psi\Big]_c = \frac{\partial\psi}{\partial n}\Big]_c = \emptyset \quad \text{(Neuman)}$$

Quindi per i <u>modi TE</u>:

$$\boxed{\nabla_t^2\psi + k_{tH}^2\psi = 0} \qquad \frac{\partial\psi}{\partial n}\Big] = 0$$

si determinano ψ e k_{tH} e quindi

V, I.

NESSUNA LIMITAZIONE SUL TIPO DI CONTORNO

$\boxed{\text{Modi TM}}$ $H_z = 0$

$$\underline{E} = \underline{E}_t + E_z \underline{i}_z = \underline{e}\, V(z) + E_z \underline{i}_z$$

$$\underline{H} = \underline{H}_t = \underline{h}\, I(z)$$

Operando come prima:

$$\begin{cases} -\dfrac{dV}{dz} = j\dfrac{k^2 - k_{tE}^2}{\omega \varepsilon} I = j\, k_{zE} Z_{0E}\, I \\ -\dfrac{dI}{dz} = j\omega\varepsilon V = j\dfrac{k_{zE}}{Z_{0E}} V \end{cases}$$

$$\underline{h} = \underline{i}_z \times \underline{e}$$

$$\nabla_t \nabla_t \cdot \underline{e} + k_{tE}^2 \underline{e} = \emptyset \qquad \underline{e} = -\nabla_t \phi$$

$$\boxed{\nabla_t^2 \phi + k_{tE}^2 \phi = \emptyset} \qquad \phi\Big]_c = \emptyset \ \text{(Dirichlet)}$$

$\Downarrow$

Infatti:

$$\underline{E} \times \underline{i_n}\Big]_c = 0 \rightarrow \left(\underline{E_t} + E_z \underline{i_z}\right) \times \underline{i_n}\Big]_c = 0$$

E_z è tutta tangente

$$\nabla_t \cdot \left(\underline{H_t} \times \underline{i_z}\right)\Big]_c = 0 \qquad I\, \nabla_t \cdot \left(\underline{h} \times \underline{i_z}\right)\Big]_c = 0$$

$$\nabla_t \cdot \left(\underline{h} \times \underline{i_z}\right)\Big]_c = \nabla_t \cdot \underline{e}\Big]_c = -\nabla^2 \phi\Big]_c = 0 \rightarrow k_{tE}^2 \phi\Big]_c = \emptyset$$

$$\boxed{\phi_c = \emptyset}$$

$$\underline{i_n} \times \underline{E_t}\Big]_c = 0 \qquad V(z)\, \underline{i_n} \times \underline{e}\Big]_c = \underline{i_n} \times \underline{e}\Big]_c =$$

$$= \underline{i_s} \cdot \underline{e}\Big]_c = -\underline{i_s} \cdot \nabla \phi\Big]_c = -\frac{\partial \phi}{\partial s}\Big]_c = \emptyset$$

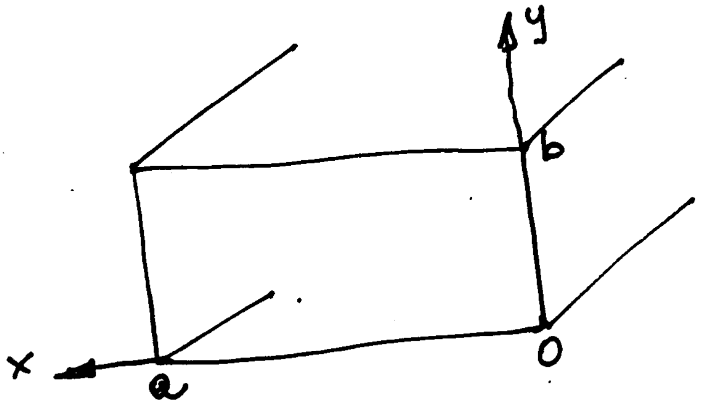
così $\boxed{\phi_c = \text{cost}}$

ϕ sul contorno deve essere cost e nulla,

quindi

$$\boxed{\phi_c = 0}$$

Esempio: Guida d'onda rettangolare.



$b = \frac{1}{2} a$ (standard

Modi TE

Equaz. trovate

$$\begin{cases} -\dfrac{dV}{dz} = j\, K_{zH} Z_{oH}\, I \\ -\dfrac{dI}{dz} = j\, \dfrac{K_{zH}}{Z_{oH}}\, V \end{cases}$$

$$\nabla_t^2 \psi + K_{tH}^2 \psi = 0$$

$$\left.\frac{\partial \psi}{\partial n}\right]_c = 0$$

$$\psi = \psi(\underline{t}) = \psi(x\, \underline{i_x} + y\, \underline{i_y}) = \psi(x,y)$$

Soluzioni del tipo: $\psi(x,y) = \psi_x(x)\, \psi_y(y)$

$$\frac{\partial^2 \psi_x}{\partial x^2} \psi_y + \frac{\partial^2 \psi_y}{\partial y^2} \psi_x + K_{tH}^2 \psi_x \psi_y = 0$$

Dividendo per $\psi_x \psi_y$ certamente $\neq 0$

$$\underbrace{\frac{1}{\psi_x} \frac{\partial^2 \psi_x}{\partial x^2}}_{\text{cost}} + \underbrace{\frac{1}{\psi_y} \frac{\partial^2 \psi_y}{\partial y^2}}_{\text{"}} + K_{tH}^2 = 0$$

$$\begin{cases} \dfrac{1}{\psi_x}\dfrac{\partial^2 \psi_x}{\partial x^2} = -k_x^2 \\ \dfrac{1}{\psi_y}\dfrac{\partial^2 \psi_y}{\partial y^2} = -k_y^2 \end{cases} \rightarrow \begin{cases} \dfrac{\partial^2 \psi_x}{\partial x^2} + k_x^2 \psi_x = 0 \\ \dfrac{\partial^2 \psi_y}{\partial y^2} + k_y^2 \psi_y = 0 \end{cases}$$

$$\boxed{k_x^2 + k_y^2 = k_{tH}^2}$$

$$\begin{cases} \psi_x(x) = A_x \cos k_x x + B_x \operatorname{sen} k_x x \\ \psi_y(y) = A_y \cos k_y y + B_y \operatorname{sen} k_y y \end{cases} \qquad \left.\frac{\partial \psi}{\partial n}\right]_c = 0$$

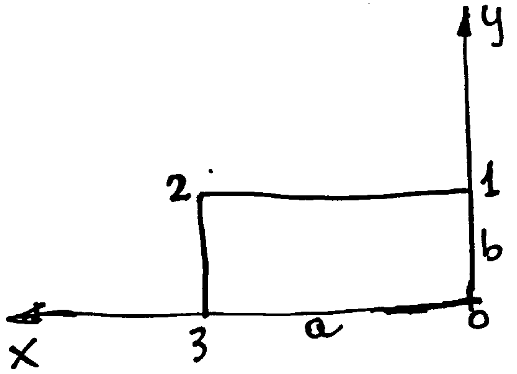
$$\left.\frac{\partial \psi}{\partial n}\right]_c = \frac{\partial}{\partial n}\left(\psi_x(x)\,\psi_y(y)\right)\Big]_c = 0$$

$$\frac{\partial \psi}{\partial x} = \psi_y \frac{\partial \psi_x}{\partial x} \quad ; \quad \frac{\partial \psi}{\partial y} = \psi_x \frac{\partial \psi_y}{\partial y}$$

$$\frac{\partial \psi_x}{\partial x} = -A_x k_x \operatorname{sen} k_x x + B_x k_x \cos k_x x$$

$$\frac{\partial \psi_y}{\partial y} = -A_y k_y \operatorname{sen} k_y y + B_y k_y \cos k_y y$$

6-25



0-1) $[x=0,\ 0\le y\le b]$ $\qquad \frac{\partial}{\partial n}\psi_x\psi_y\Big]_{x=0} = \psi_y \frac{\partial \psi_x}{\partial x}\Big]_{x=0}$

$B_x K_x \psi_y = 0$ per ogni K_x $\rightarrow \boxed{B_x = 0}$

2-3) $[x=a;\ 0\le y\le b]$ $\qquad \psi_y \frac{\partial \psi_x}{\partial x}\Big]_{x=a}$

$A_x K_x \operatorname{sen}(K_x a)\psi_y = 0$ per ogni K_x

$K_x a = m\pi \qquad \boxed{K_{xm} = \frac{m\pi}{a}}$

$(m = 0, 1, 2, \dots)$

0-3) $[0\le x\le a,\ y=0]$ $\qquad \frac{\partial}{\partial n}\psi_x\psi_y\Big]_{y=0} = \psi_x \frac{\partial \psi_y}{\partial y}\Big]_{y=0}$

$\boxed{B_y = 0}$

1-2) $[0\le x\le a,\ y=b]$ $\qquad A_y K_y \operatorname{sen}(K_y b)\psi_x = 0$

$K_{yn} = \frac{n\pi}{b} \quad (n = 0, 1, 2 \dots)$

<u>Quindi</u>, posto: $A_x A_y = A$

$$\psi_{m,n}(x,y) = \psi_{x\,m}(x)\,\psi_{y_n}(y) =$$

$$\boxed{\psi_{m,n} = A \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)}$$

$$\boxed{K^2_{t\,m,n} = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Esiste una duplice infinità numerabile di soluzioni del tipo a variabili separate. Ogni soluzione particolare $\psi_{m,n}$ (autofunzione) si associa ad uno dei valori (autovalori) che il parametro $K_{t\,m,n}$ può assumere.

$$K_{tH\,m,n} \longrightarrow K_{zH} = K^2 - K^2_{tH\,m,n}$$

$$K_{zH} \longrightarrow \psi, \quad K_{zH} \longrightarrow V_H, I_H$$

$$\psi \longrightarrow \underline{h} \longrightarrow \underline{e}$$

Vediamo le possibili configurazioni:

$m=0$
$n=0$ $\quad K_{tH\,0,0}=0 \qquad \psi = A \qquad \nabla\psi = 0$

funzioni vettoriali di modo e campi nulli.

---

modo $TE_{1,0}$

$m=1$
$n=0$ $\quad K_{tH\,1,0}=\frac{\pi}{a}$

$$\psi_{1,0} = A \cos \frac{\pi}{a} x$$

$$\underline{h} = -\nabla_t \psi = -\frac{\partial\psi}{\partial x}\underline{i_x} - \frac{\partial\psi}{\partial y}\underline{i_y} =$$

$$= A \operatorname{sen} \frac{\pi}{a} x \, \underline{i_x}$$

$$\underline{e} = \underline{h} \times \underline{i_z} = -A \operatorname{sen} \frac{\pi}{a} x \, \underline{i_y}$$

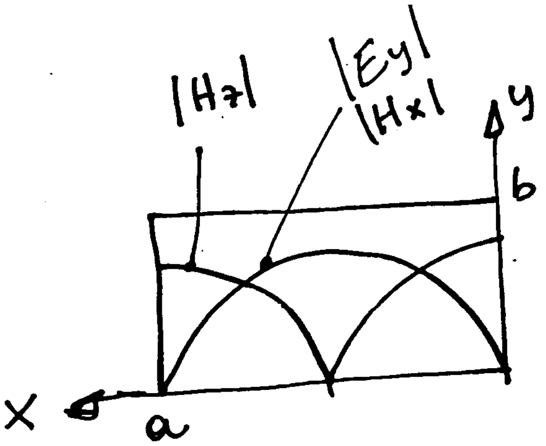
$$\underline{E_t} = -V(z) A \operatorname{sen} \frac{\pi}{a} x \, \underline{i_y}$$

$$\underline{H_t} = I(z) A \operatorname{sen} \frac{\pi}{a} x \, \underline{i_x}$$

$$H_z = \frac{1}{j\omega\mu} \nabla_t \cdot (\underline{i_z} \times \underline{E_t}) = -\frac{1}{j\omega\mu} V(z) A \nabla_t \cdot \left(\operatorname{sen}\frac{\pi}{a} x \, \underline{i_z} \times \underline{i_y}\right)$$

$$= \frac{1}{j\omega\mu} V(z) A \nabla_t \cdot \left(\operatorname{sen}\frac{\pi}{a} x \, \underline{i_x}\right) = \frac{V(z)}{j\omega\mu} \frac{A\pi}{a} \cos\frac{\pi}{a} x$$

$$\underline{H_z} = H_z \, \underline{i_z}$$



Quindi il 1° modo che si trova è

TE_{10} con $K_{t1,0} = \frac{\pi}{a}$

$TE_{2,0}$ $\quad K_{t2,0} = \frac{2\pi}{a}$

$TE_{0,1}$ $\quad K_{t0,1} = \frac{\pi}{b} > \frac{\pi}{a}$

Analogamente per modi TM:

$$\Phi_{m,n}(x,y) = B \operatorname{sen}\left(\frac{m\pi}{a}x\right) \operatorname{sen}\left(\frac{n\pi}{b}y\right)$$

con: $K^2_{tE\,m,n} = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

Se $m=0$ oppure $n=0$ → campi nulli.

Il primo modo $\neq 0$ è il

$TM_{1,1}$ con $K^2_{tE\,1,1} = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2$

TE si hanno $\infty^2 - 1$

TM si hanno $\infty^2 - 2\infty$

TEM ? è solo <u>uno</u> - Una autofunzione, un autovalore $K_t = 0$

Guida rettangolare - K_z

$$K_z^2 = K^2 - K_{t\,m,n}^2$$

Soluzione espresse come combinazione lineare di funzioni del tipo

$$\exp|\pm j K_z z|$$

→ K_z radice quadrata di K_z^2 giacente nel 4° quad

- mezzo all'interno della guida senza perdite -

Se $K^2 > K_t^2 \ldots$ K_z è reale positiva

$K^2 < K_t^2$ K_z è sull'immaginario negativo

Consideriamo:

$K^2 = K_t^2$ dove K_t deve essere del tipo:

$$K_t^2 = \omega_t^2 \varepsilon \mu$$

$$\omega_t = \frac{K_t}{\sqrt{\varepsilon \mu}} \qquad K_t = \frac{2\pi}{\lambda_t}$$

$$K_z = \sqrt{\omega^2 \varepsilon \mu - K_t}$$

Due casi: 1) $\boxed{\omega > \omega_t}$ $(\lambda < \lambda_t)$

- K_z reale positiva
- propagazione senza attenuazione

$$K_z^2 = \omega^2 \varepsilon \mu - K_t^2$$

$$\frac{K_z^2}{K_t^2} - \frac{\omega^2 \varepsilon \mu}{K_t^2} = -1$$

$$\boxed{\frac{\omega^2}{\omega_t^2} - \frac{K_z^2}{K_t^2} = 1}$$

$K_z = K_z(\omega)$ ramo di iperbole

$\omega \to \infty \qquad K_z = \omega\sqrt{\varepsilon\mu}$ (iperbole degenere per $K_t = 0$ (TEM))

2) $\omega < \omega_t \quad (\lambda > \lambda_t)$

$$K_z = \sqrt{\omega^2 \varepsilon \mu - K_t^2} = \sqrt{-(K_t^2 - \omega^2 \varepsilon \mu)} = j\sqrt{K_t^2 - \omega^2 \varepsilon \mu}$$

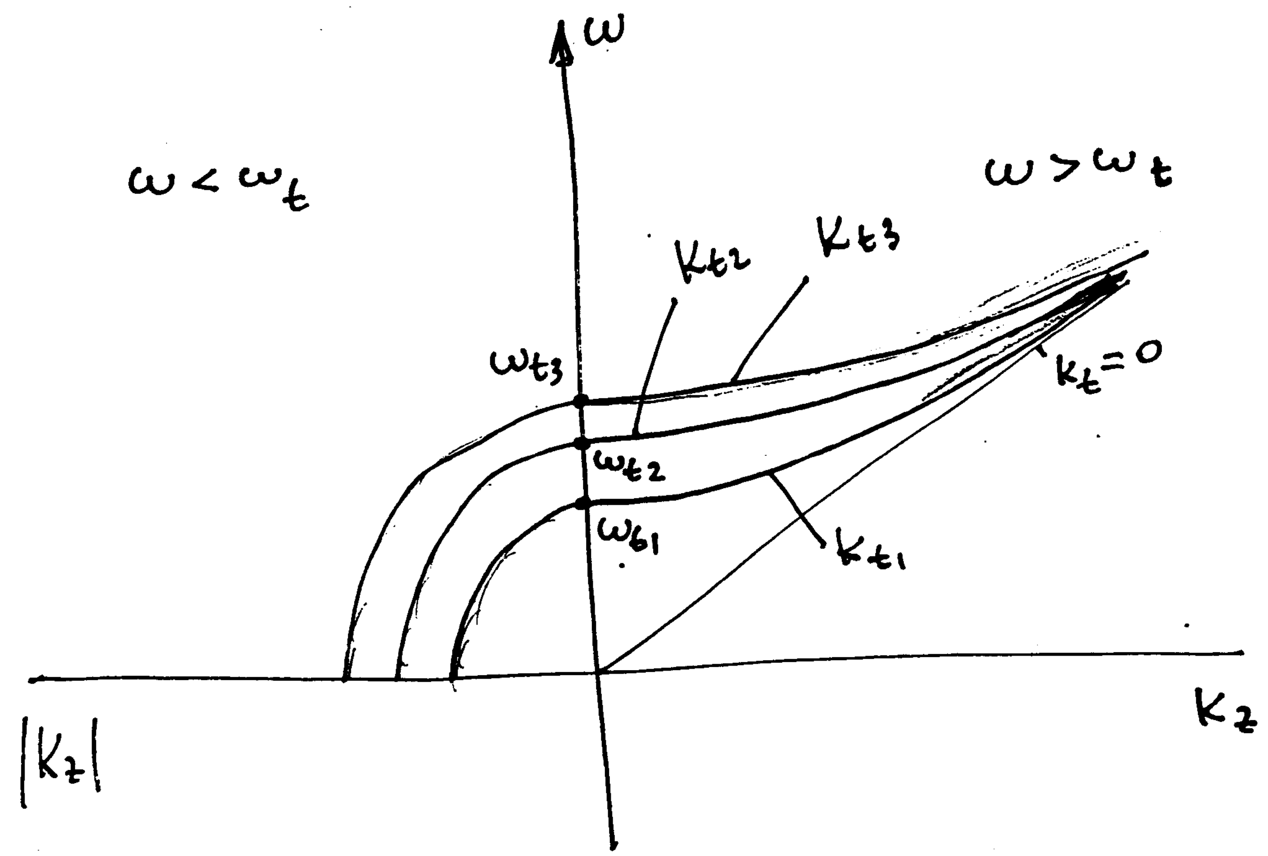
$$|K_z| = \sqrt{K_t^2 - \omega^2 \varepsilon \mu} \qquad \frac{|K_z|^2}{K_t^2} + \frac{\omega^2 \varepsilon \mu}{K_t^2} = 1$$

$$\boxed{\frac{|K_z|^2}{K_t^2} + \frac{\omega^2}{\omega_t^2} = 1}$$

$K_z = K_z(\omega)$ arco di ellisse

- K_z immaginario
- attenuazione

Riprendiamo il <u>diagramma di Brillouin</u>:



$$TE_{1,0} \quad K_{t\,1,0} = K_{t1} = \frac{\pi}{a} \qquad \lambda_{t\,1,0} = \frac{2\pi}{K_{t\,1,0}} = 2a$$

dipende <u>solo</u> dalla dimensione $\underline{\underline{a}}$

$$TE_{2,0} \quad K_{t\,2,0} = \frac{2\pi}{a} \qquad \lambda_{t\,2,0} = \frac{2\pi}{K_{t\,2,0}} = a$$

$$TE_{0,1} \quad K_{t\,0,1} = \frac{\pi}{b} \qquad \lambda_{t\,0,1} = \frac{2\pi b}{\pi} = 2b$$

se $b = \frac{a}{2}$

$$\lambda_{t\,0,1} = a$$

Modi TE — Modi TM

Impedenza caratteristica

$$Z_{0H} = \frac{\omega\mu}{k_z} \qquad\qquad Z_{0E} = \frac{k_z}{\omega\varepsilon}$$

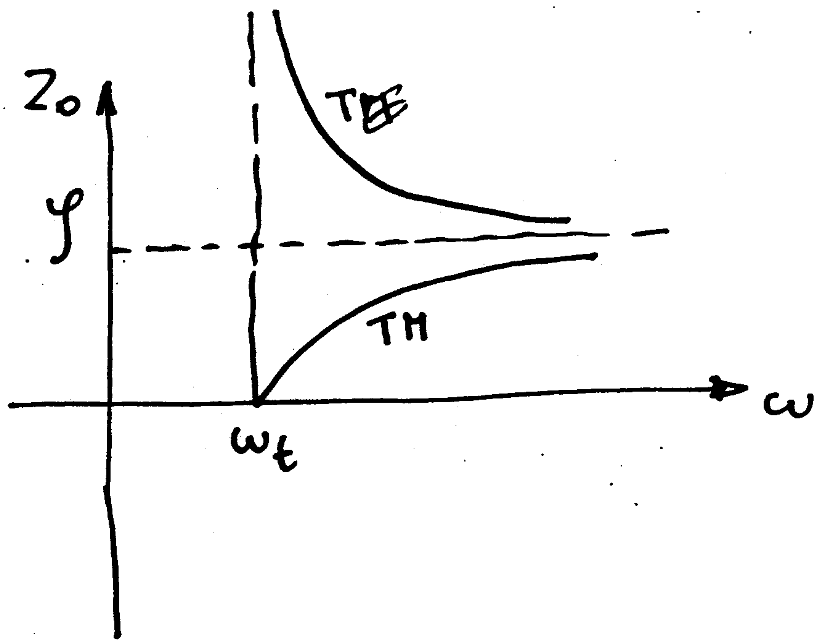
per $\omega > \omega_t$

reale e positiva

per $\omega \to \infty$ $\quad Z_{0H} = Z_{0E} = \zeta = \sqrt{\dfrac{\mu}{\varepsilon}}$

per $\omega \to \omega_t$

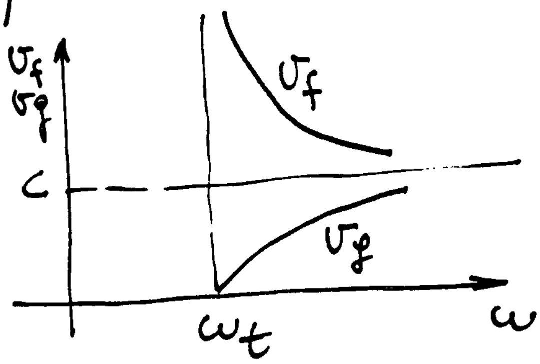
$Z_{0H} \to +\infty \qquad\qquad Z_{0E} \to 0$



$$\lambda_g = \frac{2\pi}{K_z} \qquad v_f = \frac{\omega}{K_z} \qquad v_g = \frac{d\omega}{dK_z}$$

per $\omega \to \omega_t$: $\quad \lambda_g \to \infty \qquad v_f \to \infty \qquad v_g \to 0$

per $\omega \to \infty$, $\quad K_z \to K$: $\quad \frac{2\pi}{K} \qquad \frac{\omega}{k} = c = \frac{1}{\sqrt{\varepsilon\mu}} \qquad c$



Infatti:

$$v_f = \frac{\omega}{K_z} = \frac{\omega}{\sqrt{\omega^2\mu\varepsilon - K_t^2}} \qquad v_g = \frac{d\omega}{dK_z} = \frac{1}{\frac{dK_z}{d\omega}} =$$

$$= \frac{1}{\frac{d}{d\omega}\sqrt{\omega^2\varepsilon\mu - K_t^2}} = \frac{1}{\frac{1}{\cancel{2}}\left(\omega^2\varepsilon\mu - K_t^2\right)^{-\frac{1}{2}} \cdot \cancel{2}\omega\varepsilon\mu} =$$

$$= \frac{\sqrt{\omega^2\varepsilon\mu - K_t^2}}{\omega\varepsilon\mu}$$

$$\boxed{v_f\, v_g = \frac{1}{\varepsilon\mu} = c^2}$$

RAPPRESENTAZIONE DEI CAMPI

TE) $\quad E_{zH} = \emptyset \qquad \nabla_t \cdot (\underline{H}_{tH} \times \underline{i}_z) = 0$

$$\nabla_t \cdot (\underline{h} \times \underline{i}_z) = \emptyset \qquad\qquad \nabla \times \underline{H}_{tH} = \emptyset$$

$$\downarrow$$

$$\nabla_t \cdot \underline{e} = \emptyset$$

$$\downarrow$$

$$\nabla_t \cdot \underline{E}_{tH} = \emptyset$$

TM) $\quad H_{zE} = \emptyset \qquad \nabla_t \cdot (\underline{E}_{tE} \times \underline{i}_z) = \emptyset$

$$\nabla_t \cdot (\underline{e} \times \underline{i}_z) = 0 \qquad\qquad \nabla_t \times \underline{E}_{tE} = \emptyset$$

$$\nabla_t \cdot \underline{h} = 0$$

$$\downarrow$$

$$\nabla_t \cdot \underline{H}_{tE} = \emptyset$$

$$\underline{E}_t = \underline{E}_{tE} + \underline{E}_{tH} + \underline{E}_{to} (\text{TEM})$$

$$\underline{H}_t = \underline{H}_{tE} + \underline{H}_{tH} + \underline{H}_{to} (\text{TEM})$$